# APPLICATION OF THE FORMALIZED CIRCUITS

Nikolay Raychev

**Abstract** - The purpose of this article is to analyze the practical applications of the reconfigurable formalized circuits of Raychev, as well as to analyze the ability of the defined by the author formalized computing models for performing of scientific computations.

**Key words:** Quantum computing, circuit, operators, gates

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## 1. INTRODUCTION

The focus of this Article is to be demonstrated two model of universal quantum computations with the formalized circuits of Raychev, which provide an architecture that can be used for appropriate algorithmic reprogramming of quantum schemes. The size and design of the architecture are fixed, but it itself is scalable, which makes it re-programmable, flexible and universal. In a purely quantum computer there is no other such fixed quantum logic architecture, which can be realized in deterministic type. This is because the dimension of the formalized programming system is infinite, which means that each set of qubit operations is endless. The control of the formalized system could be supported by a classical computer.

Here we present details on the implementation of the steps of the input modification, the formation ($V_f$) and the combination ($V_c$). A plan of the matrix format of the operations can be found in equation (A3) - for the cases with one qubit in the first circuit model - and equation (A8) and equation (A9) - for the cases with two qubits in the second circuit model; here the empty spaces denote zeros, and the points - matrix parts, which are of no interest for the final operation.

## 2. THE CIRCUITS
### a. First circuit model

Starting with random input data $|\psi\rangle = (\alpha_0, \alpha_1)^T$ and the following arbitrary unitary matrix:

$$U = \begin{pmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{pmatrix} \qquad (A1)$$

the first method requires $2n+1 = 3$ qubits for simulation with the input data:

$$|\psi_{initial}\rangle = |0\rangle \otimes |0\rangle \otimes |\psi\rangle = \begin{pmatrix} a_0 \\ a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (A2).$$

The subsequent represent a matrix $V_f$, the matrix after the step of combination $V$ and modified input data $|\tilde{\psi}\rangle$:

$$V_f = \begin{pmatrix} u_{00} & \cdot & & & & & \\ \cdot & \cdot & & & & & \\ & u_{01} & \cdot & & & \\ & & \cdot & \cdot & & \\ & & & u_{10} & \cdot & \\ & & & & \cdot & \cdot \\ & & & & & u_{11} & \cdot \\ & & & & & & \cdot \end{pmatrix}, \qquad V = \frac{1}{\sqrt{2}} \begin{pmatrix} u_{00} & \cdot & u_{01} & \cdot & & & & \\ \cdot & \cdot & \cdot & \cdot & & & & \\ \cdot & \cdot & \cdot & \cdot & & & & \\ \cdot & \cdot & \cdot & \cdot & & & & \\ & & & & u_{10} & \cdot & u_{11} & \cdot \\ & & & & \cdot & \cdot & \cdot & \cdot \\ & & & & \cdot & \cdot & \cdot & \cdot \\ & & & & \cdot & \cdot & \cdot & \cdot \end{pmatrix},$$

$$|\psi\rangle = \begin{pmatrix} a_0 \\ 0 \\ a_1 \\ 0 \\ a_0 \\ 0 \\ a_1 \\ 0 \end{pmatrix} \qquad (A3).$$

In order to illustrate, below we represent complete forms of some of the operators and output vector for the same case:

The complete form of the obtained matrix from the step of formation is as follows:

$$\begin{matrix} u_{00} & \sqrt{1-u_{00}^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{1-u_{00}^2} & u_{00} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & u_{01} & \sqrt{1-u_{01}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{1-u_{01}^2} & u_{01} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & u_{10} & \sqrt{1-u_{10}^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{1-u_{10}^2} & u_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & u_{11} & \sqrt{1-u_{11}^2} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{1-u_{11}^2} & u_{11} \end{matrix}$$

$$(A4).$$

The combined matrix $V_c$ and the matrix for the input modification $V_M$ are defined as:

$$V_c = \begin{matrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{matrix}, V_m = \begin{matrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} \end{matrix}$$

$$(A5).$$

For the initial input data $|\psi_{initiali}\rangle$, as in equation (A2), the final output state $|\psi_{finali}\rangle$ becomes:

$$|\psi_{final}\rangle = V_c V_f V_m |\psi_{initial}\rangle$$

$$= \frac{1}{2}\begin{pmatrix} a_0 u_{00} + a_1 u_{01} \\ -a_0\sqrt{1-u_{00}^2} - a_1\sqrt{1-u_{01}^2} \\ a_0 u_{00} - a_1 u_{01} \\ -a_0\sqrt{1-u_{00}^2} + a_1\sqrt{1-u_{01}^2} \\ a_0 u_{10} + a_1 u_{11} \\ -a_0\sqrt{1-u_{10}^2} - a_1\sqrt{1-u_{11}^2} \\ a_0 u_{10} - a_1 u_{11} \\ -a_0\sqrt{1-u_{10}^2} + a_1\sqrt{1-u_{11}^2} \end{pmatrix} \qquad (A6).$$

Apparently the normalized states $|00 - 0\rangle$ and $|10 - 0\rangle$ simulate the initial given system.

### b.  Second circuit model

For the same case, because the second circuit model initially works with at least one pair of matrix elements, it will create an unitarity in the initial step. There is no need of a step for combination. So the result will be simulated on states $|00\rangle$ and $|10\rangle$. For the two qubit systems below, the simulation goes, as follows:

$$U = \begin{pmatrix} u_{00} & u_{01} & u_{02} & u_{03} \\ u_{10} & u_{11} & u_{12} & u_{13} \\ u_{20} & u_{21} & u_{22} & u_{23} \\ u_{30} & u_{31} & u_{32} & u_{33} \end{pmatrix} \qquad (A7).$$

In the step of formation, if we use blocks 4 by 4, as shown in Fig. 4, the step of combination will not be needed, because we will already have formed rows of $U$ in the step of formation. However, if we use 2 by 2 initial blocks, we need to use one rotational gate for each pair of elements, then - step for combination. Thus, at the end of the formation step, we obtain the following matrix:

$$V_f = \begin{pmatrix} k_0 u_{00} & k_0 u_{01} \\ & & k_1 u_{02} & k_1 u_{03} \\ & & & & k_2 u_{10} & k_2 u_{11} \\ & & & & & & k_3 u_{12} & k_3 u_{13} \\ & & & & & & & & k_4 u_{20} & k_4 u_{21} \\ & & & & & & & & & & k_5 u_{22} & k_5 u_{23} \\ & & & & & & & & & & & & k_6 u_{30} & k_6 u_{31} \\ & & & & & & & & & & & & & & k_7 u_{32} & k_7 u_{33} \end{pmatrix}$$

$$(A8),$$

where $k_i$ are normalization constants. After subsequent steps of combination and modification of the input data we obtain the following matrices and modification of the input data:

$$V = \begin{pmatrix} u_{00} & u_{01} & u_{02} & u_{03} & & & & & & & & \\ \cdot & \cdot & \cdot & \cdot & & & & & & & & \\ \cdot & \cdot & \cdot & \cdot & & & & & & & & \\ \cdot & \cdot & \cdot & \cdot & & & & & & & & \\ & & & & u_{10} & u_{11} & u_{12} & u_{13} & & & & \\ & & & & \cdot & \cdot & \cdot & \cdot & & & & \\ & & & & \cdot & \cdot & \cdot & \cdot & & & & \\ & & & & \cdot & \cdot & \cdot & \cdot & & & & \\ & & & & & & & & u_{20} & u_{21} & u_{22} & u_{23} \\ & & & & & & & & \cdot & \cdot & \cdot & \cdot \\ & & & & & & & & \cdot & \cdot & \cdot & \cdot \\ & & & & & & & & \cdot & \cdot & \cdot & \cdot \\ & & & & & & & & & & & & u_{30} & u_{31} & u_{32} & u_{33} \\ & & & & & & & & & & & & \cdot & \cdot & \cdot & \cdot \\ & & & & & & & & & & & & \cdot & \cdot & \cdot & \cdot \\ & & & & & & & & & & & & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$|\tilde{\psi}\rangle = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \qquad \text{(A9).}$$

The final step is equivalent to $|\psi_{finali}\rangle = V |\tilde{\psi}\rangle$. In $|\psi_{finali}\rangle$, the states $|0000\rangle$, $|0100\rangle$, $|1000\rangle$ and $|1100\rangle$ are respective states that simulate initially the unitary matrix.

## Annex B: Explicit circuit for unitary propagator of a hydrogen molecule

As it was mentioned, the unitary matrix $U_{H2}$ for the hydrogen molecule has 19 non-null elements, 15 of which are localized on the diagonal. Because the unitary matrix is 16 by 16, we need 4 main and 5 ancilla qubits for the first circuit model, given in Fig. 3. And the constantly controlled rotational gates in the step for formation are the gates $R_y$ gates, followed by the $R_z$ gates, where we use an identity for the non-zero elements. We, however, can take advantage of the sparsity of the matrix and to reduce the number of the ancilla qubits to 2 instead of 5: The non-diagonal matrix elements are localized at (13, 4), (4, 13), (7, 10) and (10, 7), where (*i, j*) are indices for the rows and columns. We apply a matrix for rearrangement (permutation) *P*, in order to reduce the range of the matrix. $PU_{H2}$ takes non-diagonal elements (13, 4), (4, 13), (7, 10) and (10, 7) to (5, 4), (4, 5), (7, 8) and (8, 7), which creates
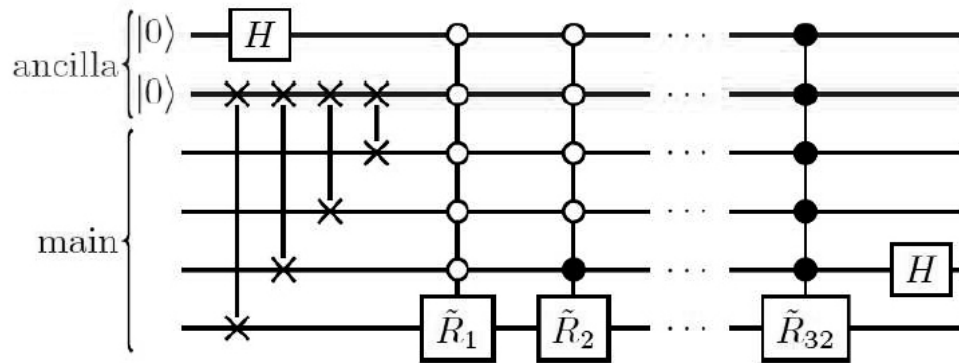


another unitary circuit~$U_{H2}$ . ~$U_{H2}$ is a structured matrix, where all the elements are localized in (*i, i*), (i, *i + 1*) or (*i − 1, i*) positions. Thus we can use 2 qubits for the ancilla and 4 qubits for the main ones, in order to create a matrix *V*, having 4 × 4 block matrices on its diagonal through the use of only a single gate of Hadamard in the combination step. In the step for formation, the control qubits for $R_y$ gates and $R_z$ gates are defined, in order to form the couples (*i, i*) and (*i, i + 1*), or (*i − 1, i*) and (*i, i*) elements on the first row of these 4 by 4 matrices. The values of the angles are defined by the polar representation of each element and are given in Table I. The circuit for ~$U_{H2}$ is shown in Fig. 8, where ~*R* represents a combination of gates $R_y$ and $R_z$. Please note that the circuit equivalents of the matrices for permutation such as *P* are combinations of the multi control CNOT gates, where which elements will be transferred is defined by the control qubits. And the input data should also be permuted before the circuit. This can be done simply by transferring the input data for the qubits. At the end of this circuit, because the leading rows of 4 by 4 matrices simulate unitary matrix, we receive simulation result from the states **|0›, |4›, |8›, |12›, . . . , |60›**.

*Fig. 8: Circuit for simulation of the hydrogen molecule.*

The values of the angles for the rotational gates are defined in order to be created the elements of ~$U_{H2}$: There are only 19 rotational gates, the remaining are *X* gates, in order to be received the right order of the elements after the combination. For the diagonal elements of ~$U_{H2}$ these rotations are only around the *z*-axis. For the non-null diagonal elements the rotation around the *z*-axis is followed by rotations around the *y* axis. The angles for these gates are given in Table I

| State of Control Qubits | Matrix Elements | Angle for $R_z$ | Angle for $R_y$ |
|---|---|---|---|
| 00000 | 0.9788-0.2049i | -0.4127 | 0 |
| 00010 | 0.3987+0.9171i | 2.3214 | 0 |
| 00100 | 0.3987+0.9171i | 2.3214 | 0 |
| 00110 | -0.2607+0.9517i | 3.6763 | 0.3253 |
| 00111 | 0.1401-0.0817i | -1.0559 | 2.8158 |
| 01000 | 0.1401-0.0817i | -1.0559 | 2.8158 |
| 01001 | -0.2607+0.9517i | 3.6763 | 0.3253 |
| 01011 | 0.9354+0.3535i | 0.7226 | 0 |
| 01101 | 0.3189+0.9478i | 2.4925 | 0 |
| 01110 | 0.4766+0.8604i | 2.1299 | 0.3629 |
| 01111 | -0.1577+0.0874i | 5.271 | 2.779 |
| 10000 | -0.1577+0.0874i | 5.271 | 2.779 |
| 10001 | 0.4766+0.8604i | 2.1299 | 0.3629 |
| 10011 | 0.3130+0.9498i | 2.5049 | 0 |
| 10101 | 0.3189+0.9478i | 2.4925 | 0 |
| 10111 | 0.3130+0.9498i | 2.5049 | 0 |
| 11001 | 0.9569+0.2410i | 0.4934 | 0 |
| 11011 | 0.8889+0.4582i | 0.9519 | 0 |
| 11101 | 0.8889+0.4582i | 0.9519 | 0 |
| 11111 | 1 | 0 | 0 |

*TABLE I: Parameters for the rotational gates*

## 3. CONCLUSION

It is important to note that, in this work, although we have in mind real matrices, it is not difficult to realize each complex case both through consideration of each rotational gate as capable to produce each complex element of an unitary matrix in the first model of a circuit. This may require more than one conventional rotational gate, but should not increase the upper limit of the quantum complexity. The modification of the second circuit, however, may not be so simple, as in the first one: it may require additional gates during the steps for combination and formation.

**Finding angles**

Upon searching the values of the angles with classic computers for a given unitary operator the process can be paralelized conveniently to find the angles. For example, the distribution of each row to different cores may be a manner of parallelization of the method. This may further be improved and designed with a view of more small blocks. And so the time for computation for generation of the angles for both circuits can be very brief.

The procedure for combination, described for both process of designing, is possible to be further improved in order to combine the circuits for different unitary operations, by accepting them for initial blocks. One of the blocks, which are used for generation of a row of the given matrix can also be used as a circuit for preparing the states (e.g. Fig. 7) for an arbitrary circuit. Moreover, the circuits, which are generated by the first method, have a high similarity with qubus quantum computer [22]. It is also possible similar ideas to be used for implementation of techniques for circuit models for this type of quantum computers.

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